NUI GALWAY–UL ALLIANCE

5TH RD

5TH POSTGRADUATE RESEARCH DAY 2015

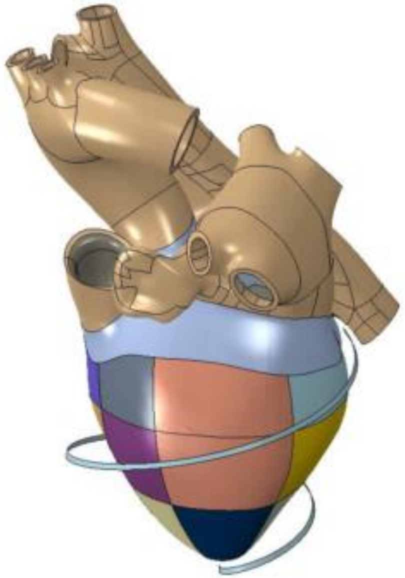
Ilaria Cinelli

Thermo-electrical equivalents for simulating the electro-mechanical behavior of biological tissue

NUI Galway OÉ Gaillimh

BMEC Biomechanics Research Centre

University of Limerick Ollscoil Luimnigh



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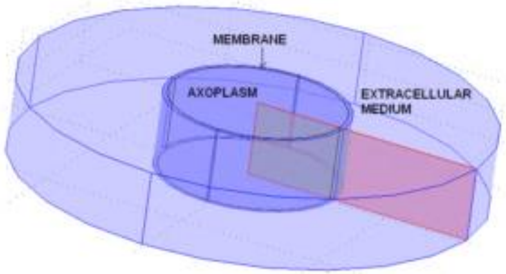

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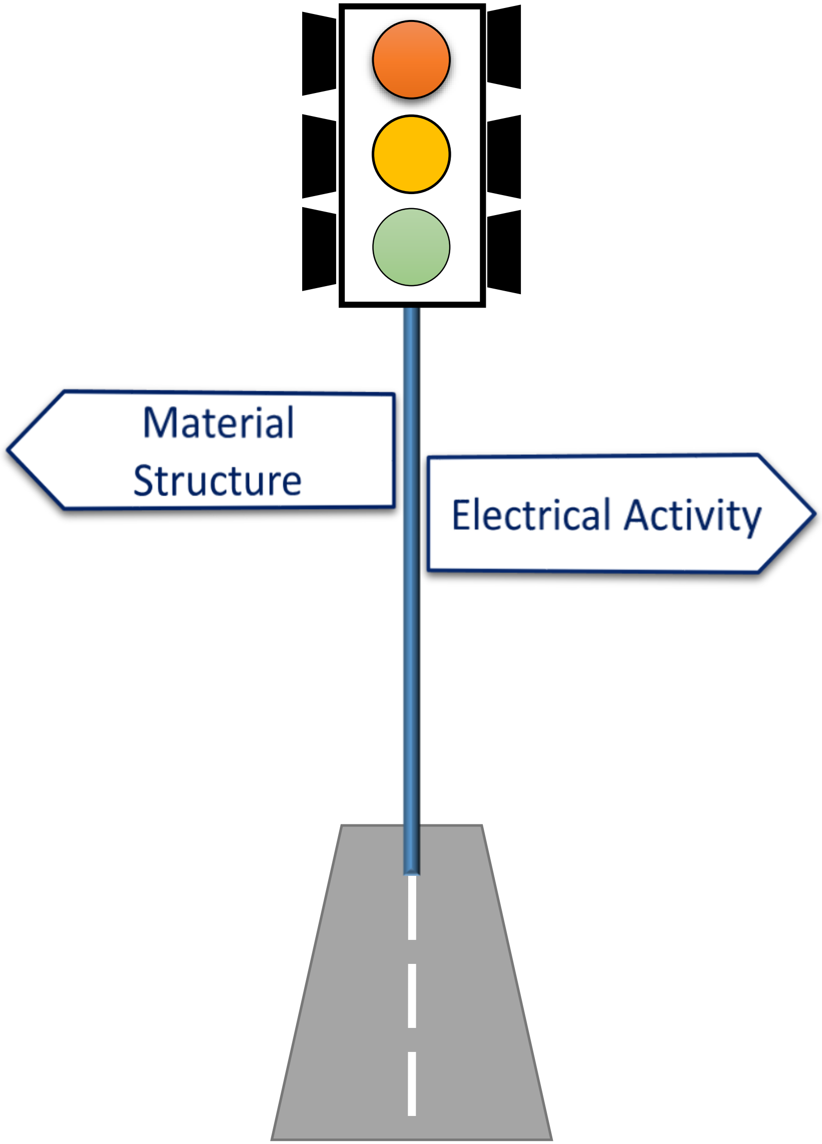

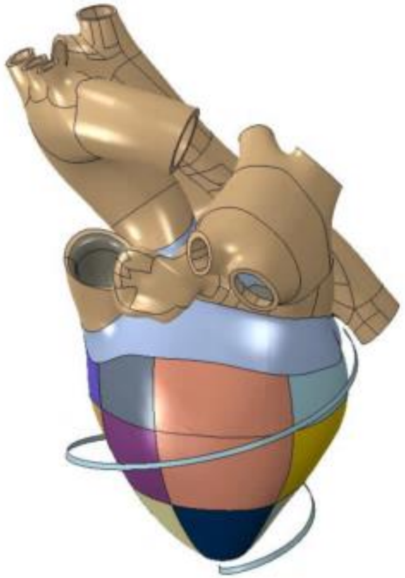

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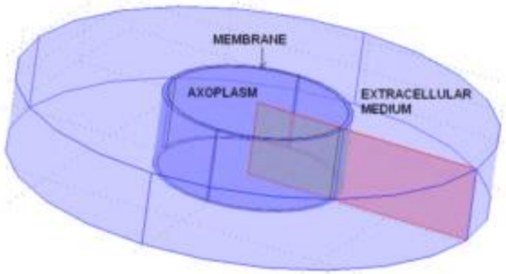

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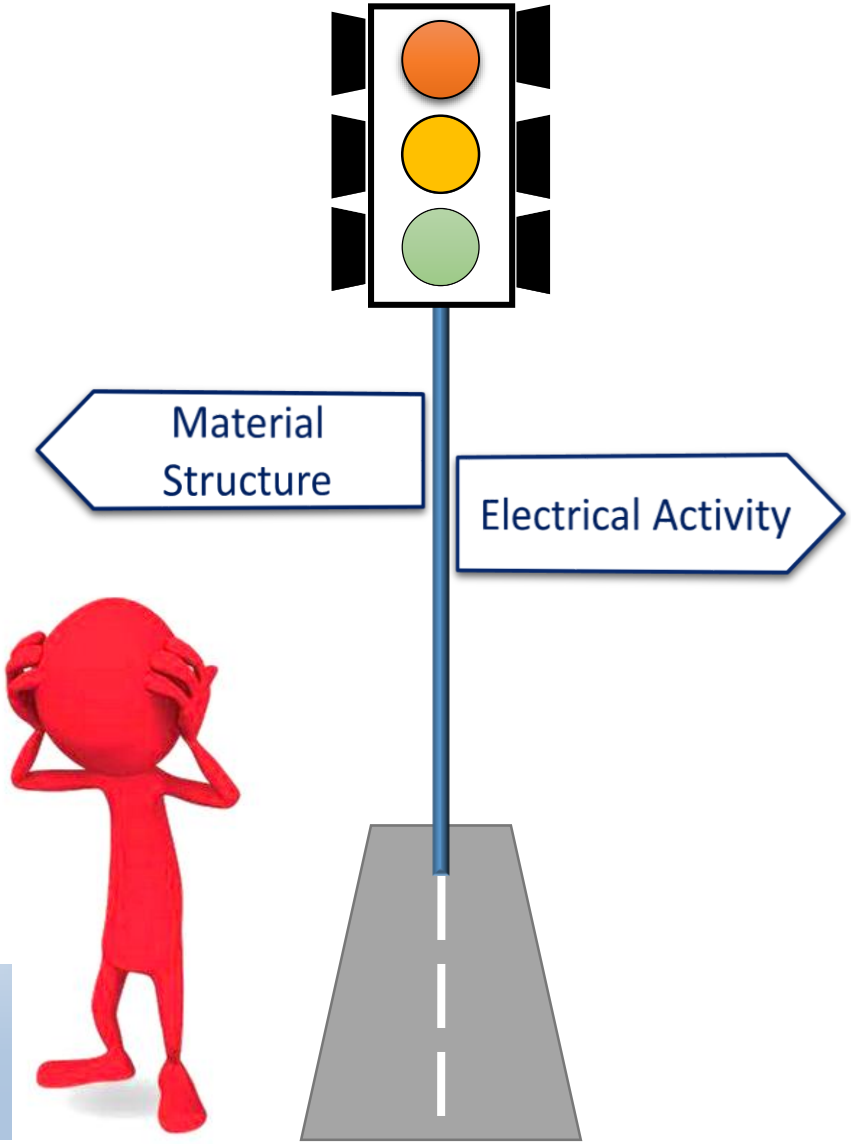
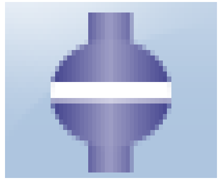
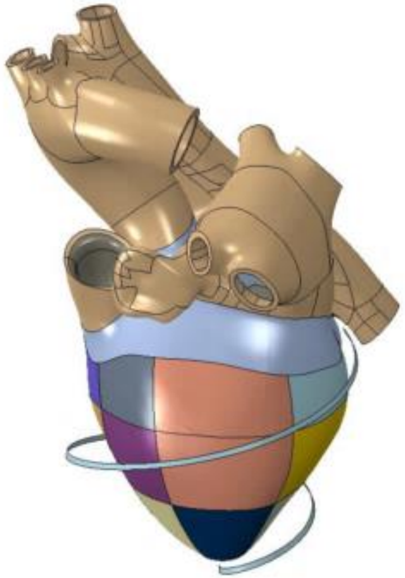


6.13-3





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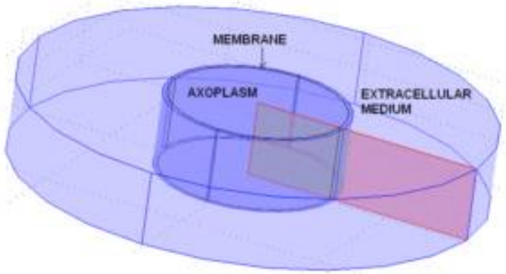

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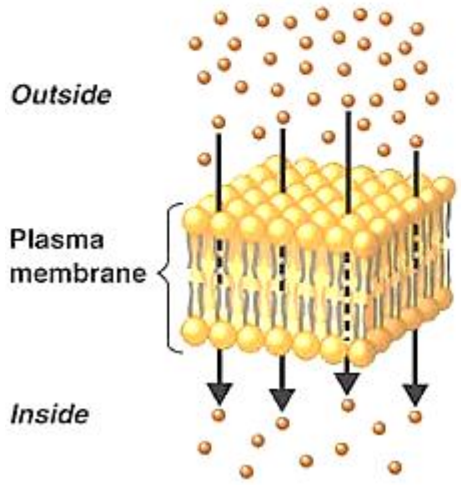

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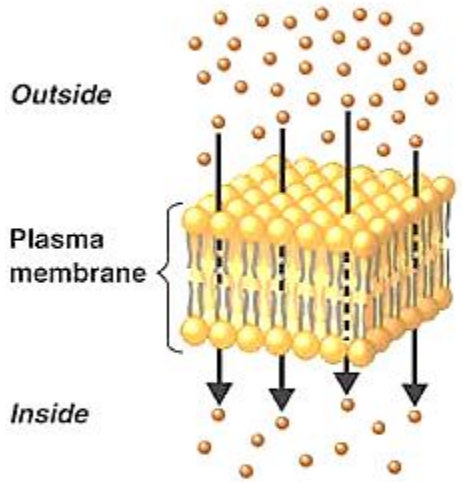

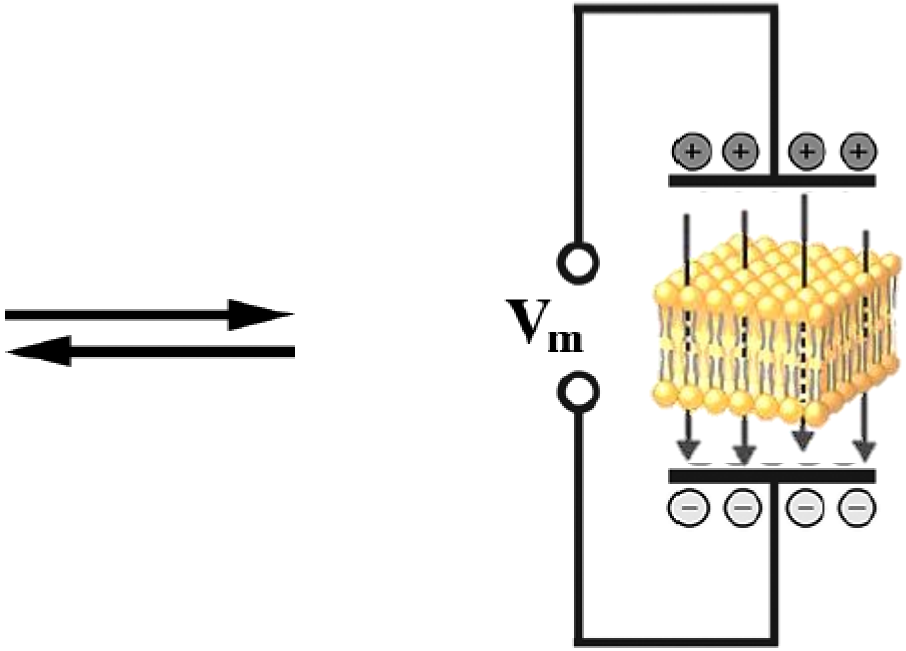

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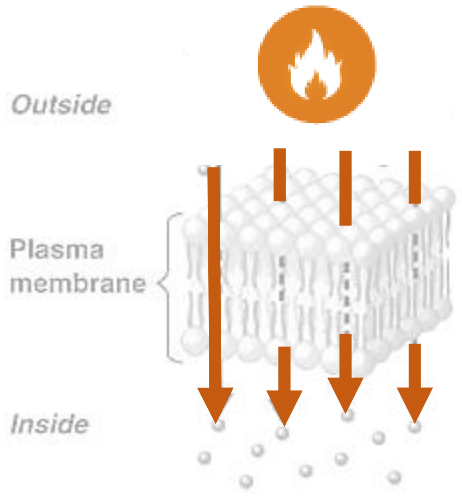

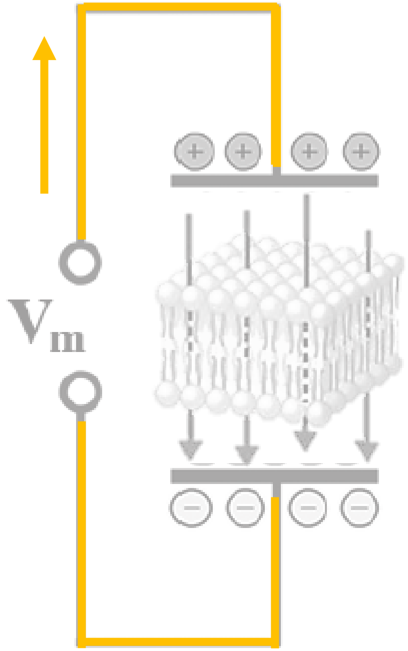

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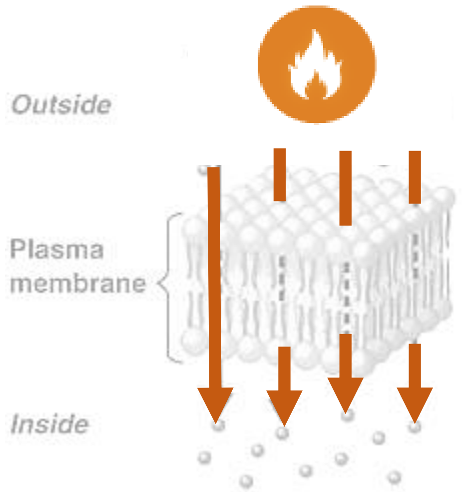
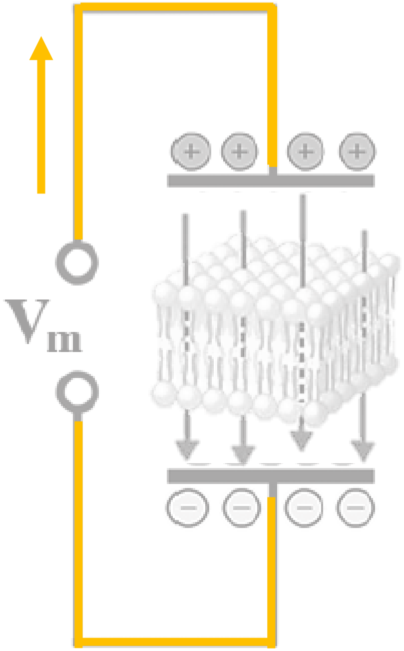
$[Watt]$





$[Ampere]$

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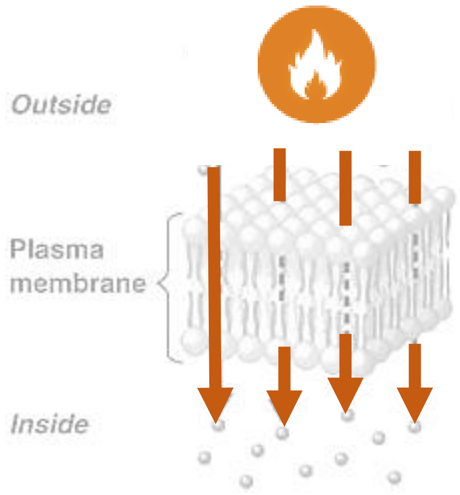
[*Watt*]



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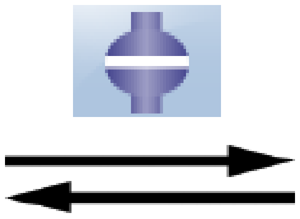
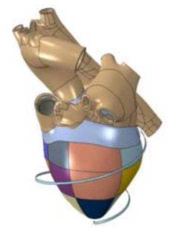


[1],[4],[7]*



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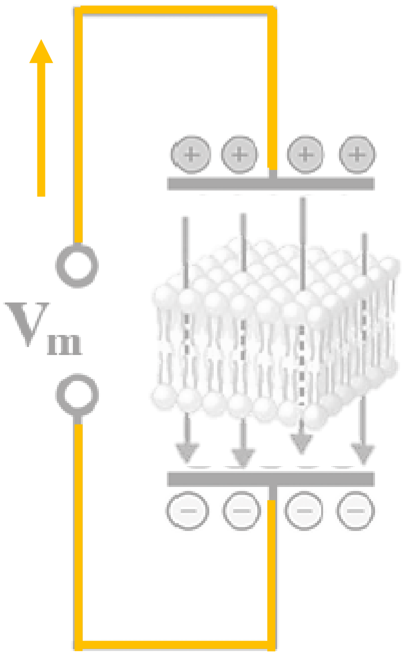

[*Ampere*]

** Quasi-static theory*

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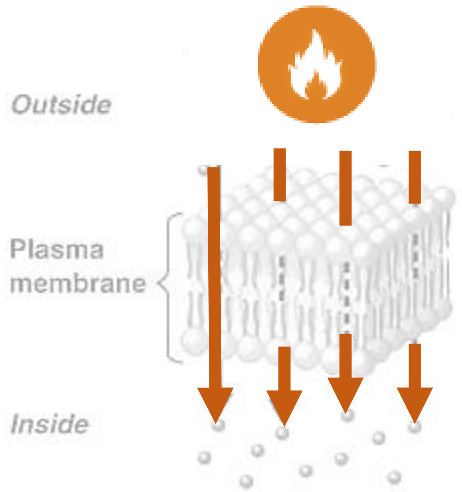

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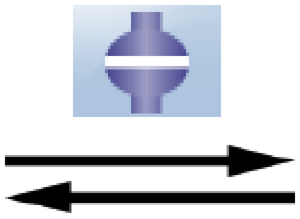

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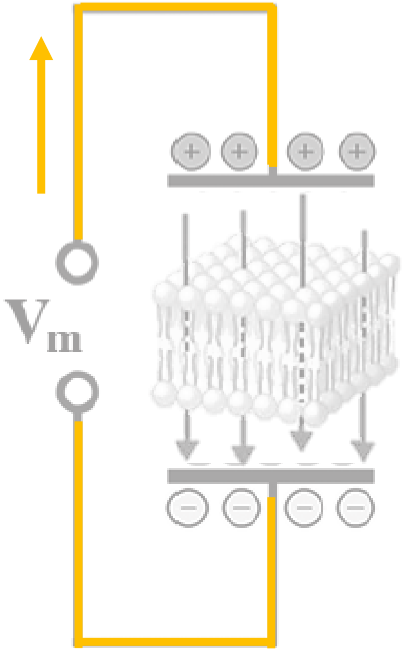

$[Ampere]$

Temperature	[°C]	Voltage	[V]
Joule	[J]	Coulomb	[C]

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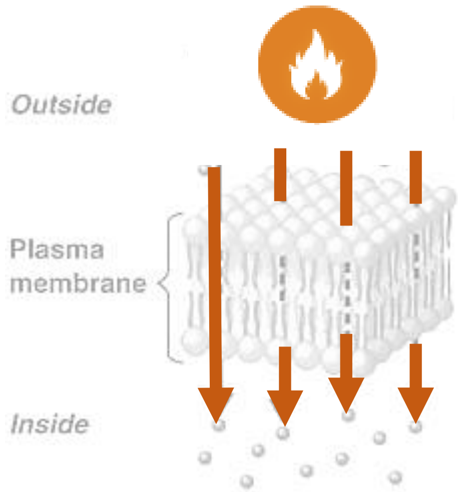
$[Watt]$



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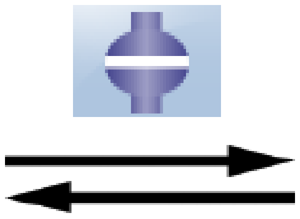

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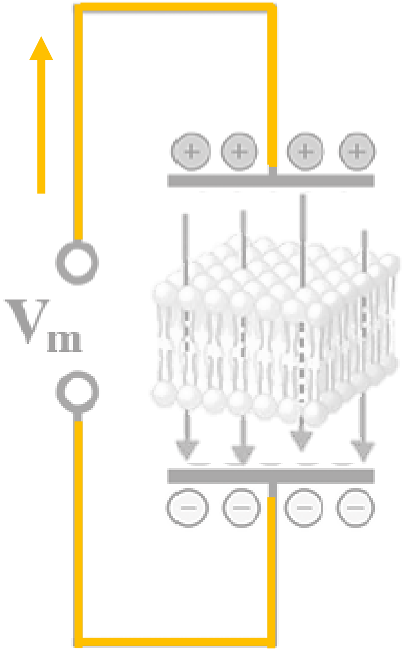

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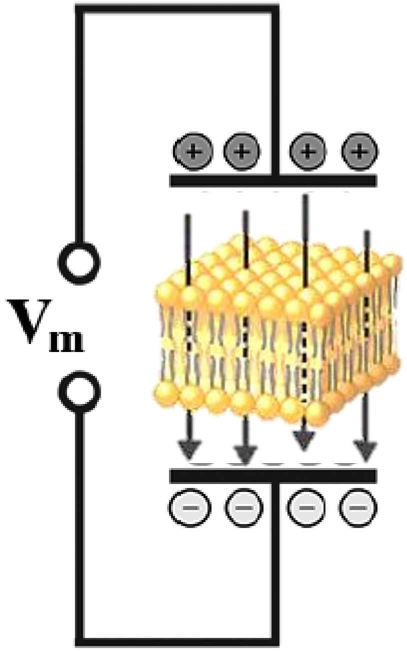
Steady State

$$\nabla \cdot k\nabla T + Q = 0 \qquad [^{W}/_{m^3}]$$

$$\nabla \cdot \sigma\nabla V + \rho\ ^{\sigma}/_{\varepsilon} = 0 \qquad [^{A}/_{m^3}]$$



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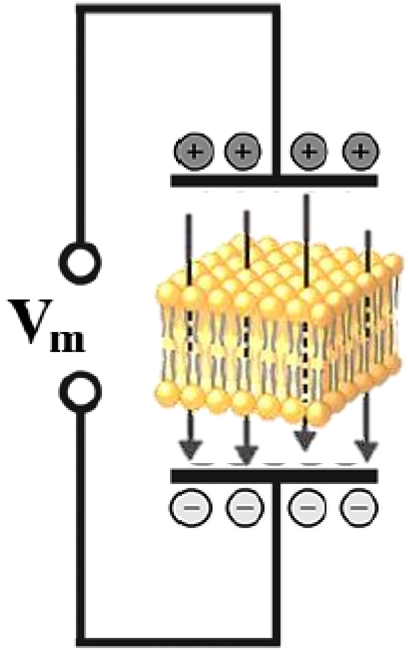

Transient

$$\rho c_p \frac{\partial T}{\partial t} - \nabla \cdot k\nabla T - Q = 0 \qquad [{}^{W}/_{m^3}]$$

$$C_m S_v \frac{\partial V_m}{\partial t} - \frac{1}{R_i}\nabla^2 V_m + S_v I = 0 \qquad [{}^{A}/_{m^3}]$$ [5]

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Steady State

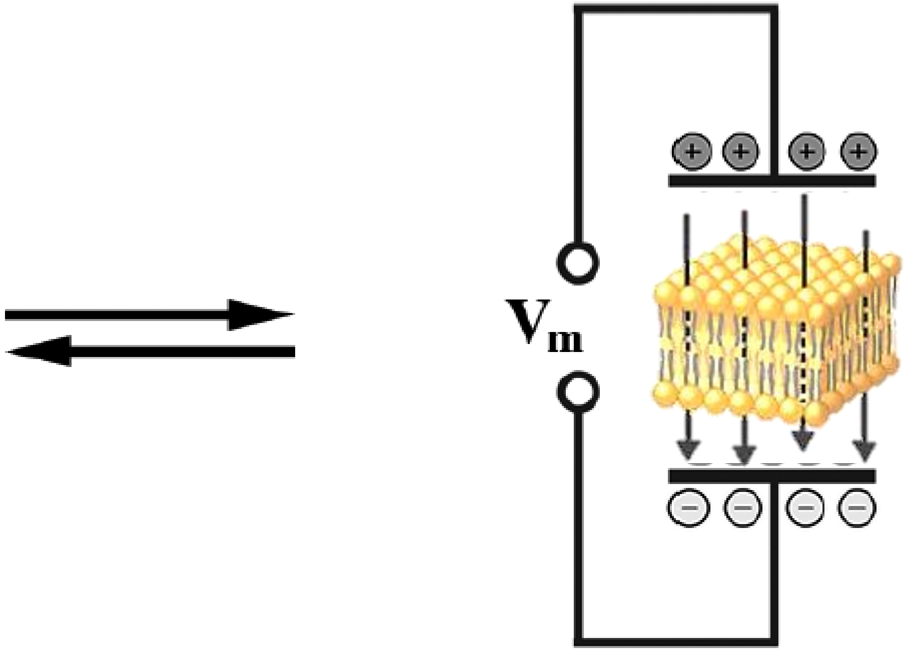
$$\nabla \cdot k\nabla T + Q = 0 \qquad [W/m^3]$$

$$\nabla \cdot \sigma\nabla V + \rho\, \sigma/\varepsilon = 0 \qquad [A/m^3]$$

Transient

$$\rho c_p \frac{\partial T}{\partial t} - \nabla \cdot k\nabla T - Q = 0 \qquad [W/m^3]$$

$$C_m S_v \frac{\partial V_m}{\partial t} - \frac{1}{R_i}\nabla^2 V_m + S_v I = 0 \qquad [A/m^3]$$ [5]



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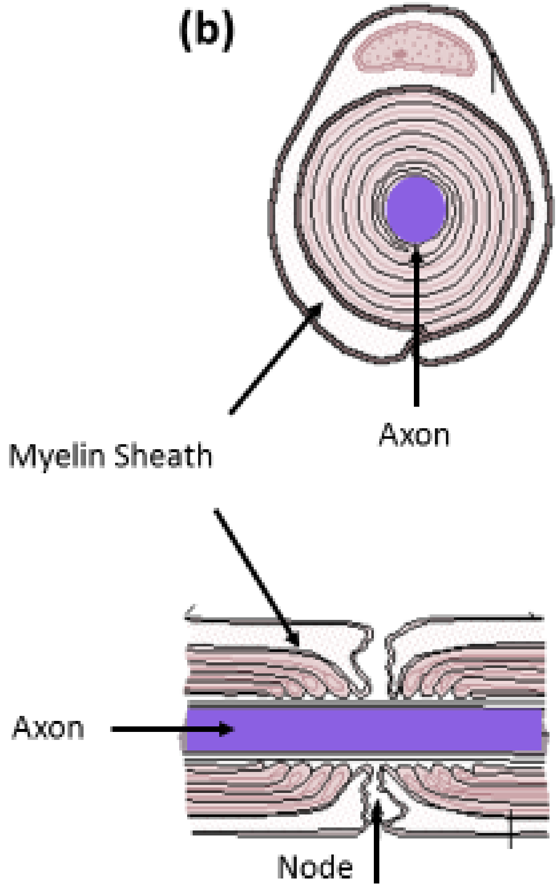

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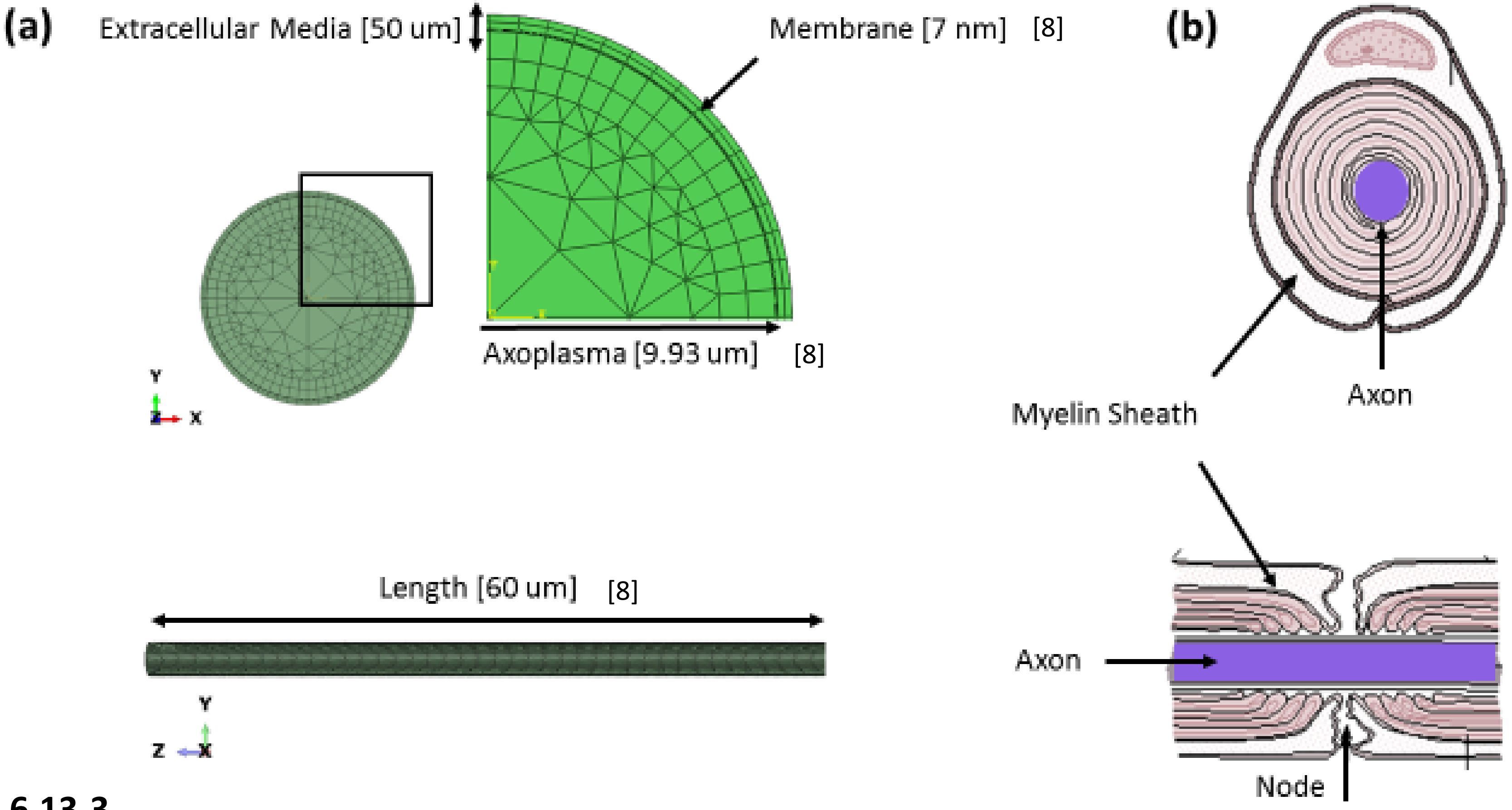

6.13-3

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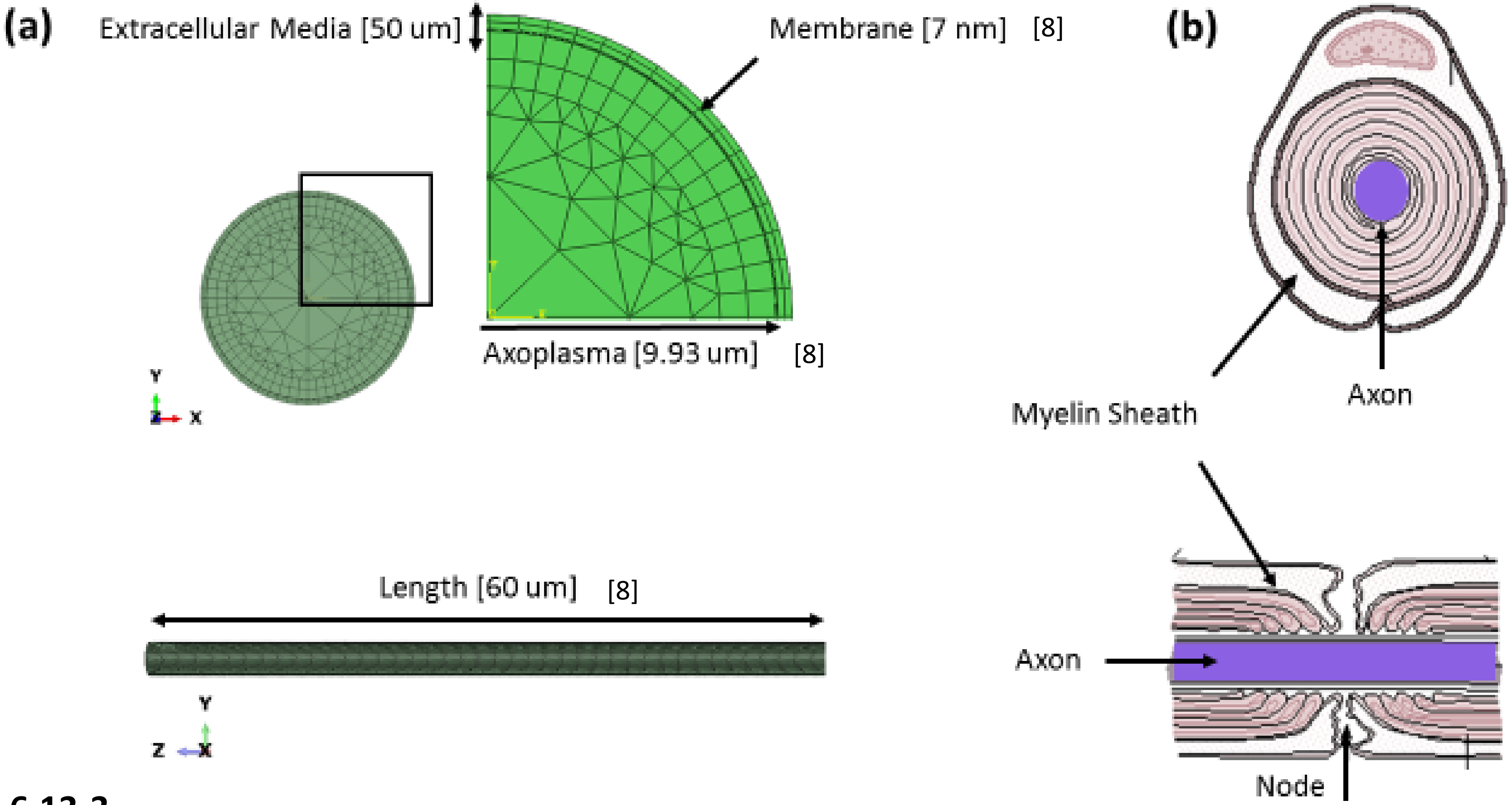
Material Properties	Units
Conductivity	$\mathrm{S\ um^{-1}}$
Specific Heat	$\mathrm{F\ um^{-2}}$
Density	$\mathrm{kg\ um^{-3}}$



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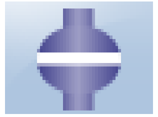

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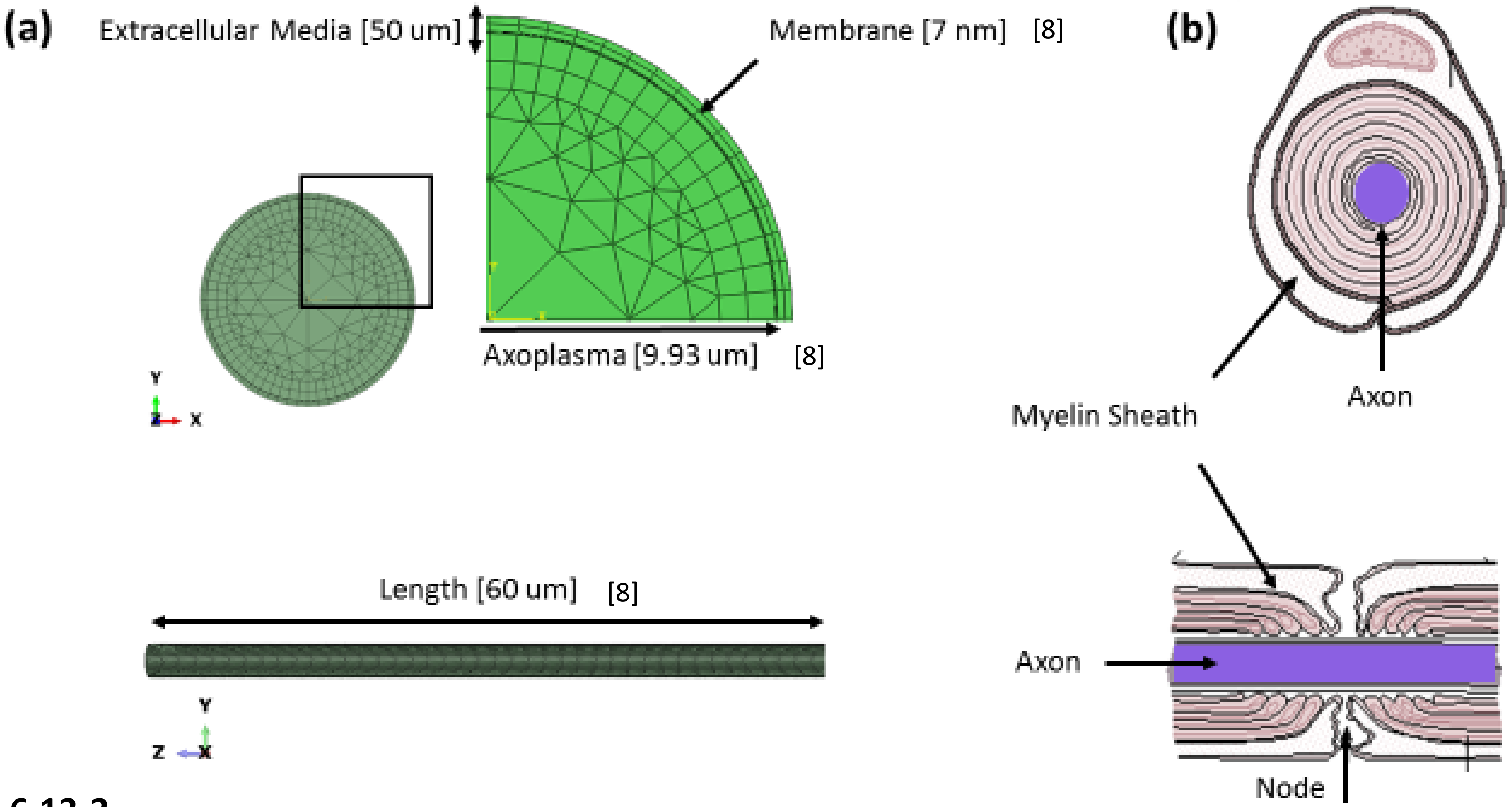
[8] Meffin, H., (2012).

- Isotropic 3D cylindrical model ;
- Number of nodes: 310717;
- Number of elements: 350400;
- Element Types: DC3D6,DCC3D8.



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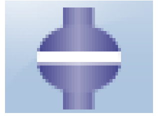

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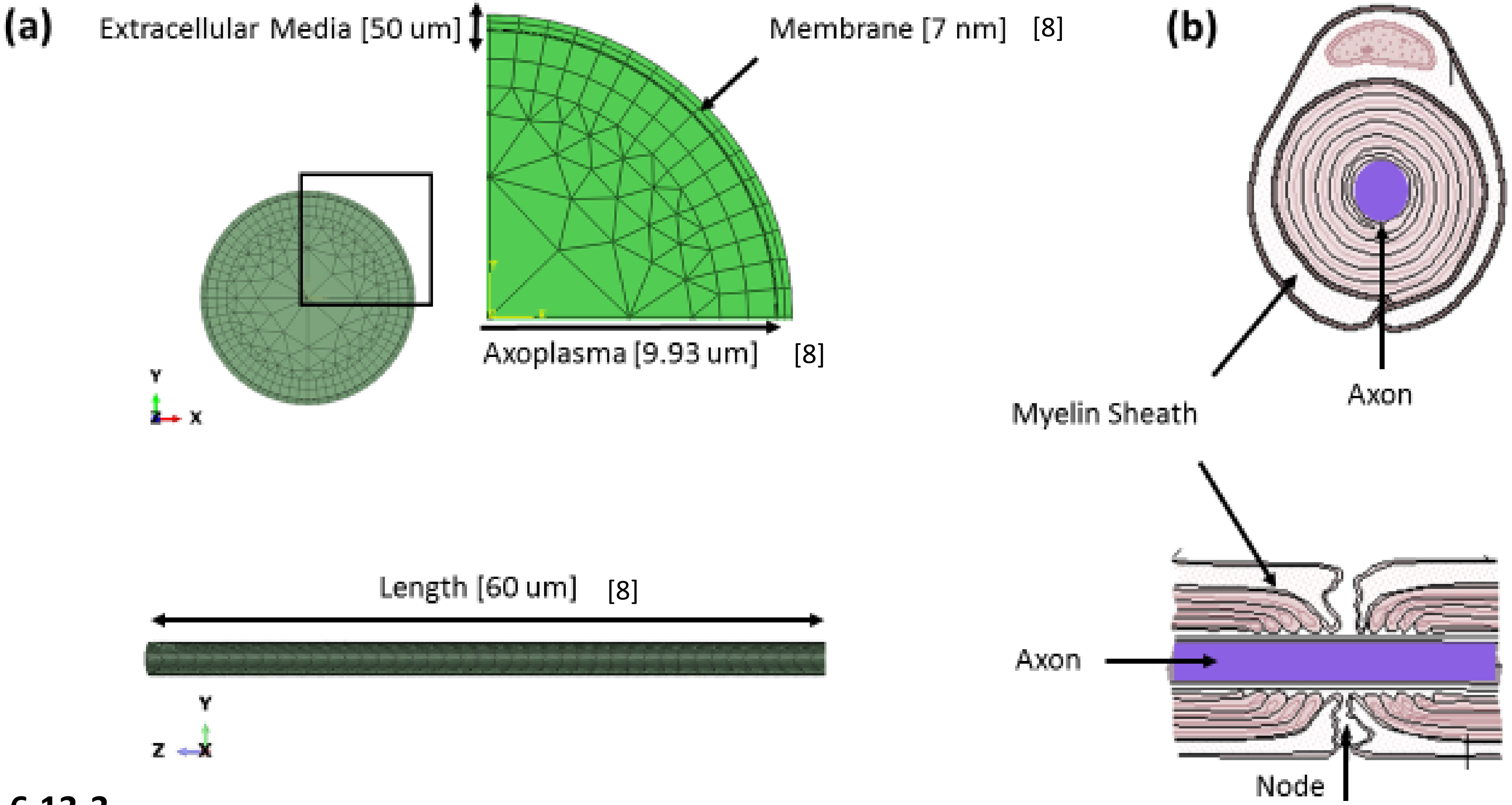
- ✓ Boundary Conditions [3],[8]

$$\begin{cases} \frac{\partial V}{\partial x}(x_0,t)=0 \\ V(x_L,t)=0 \end{cases} \quad \begin{cases} \frac{\partial V}{\partial x}(y_0,t)=0 \\ \frac{\partial V}{\partial x}(y_L,t)=0 \end{cases}$$

- ✓ Load

$$V(x_m,t)=DISP$$



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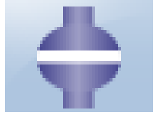

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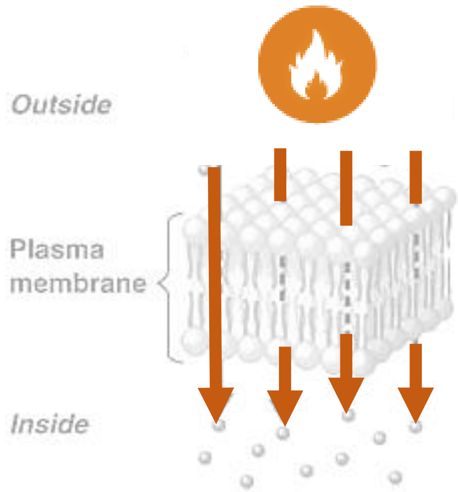

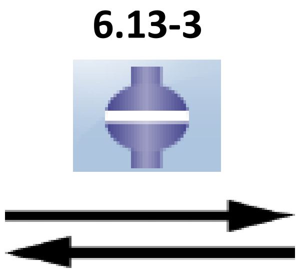

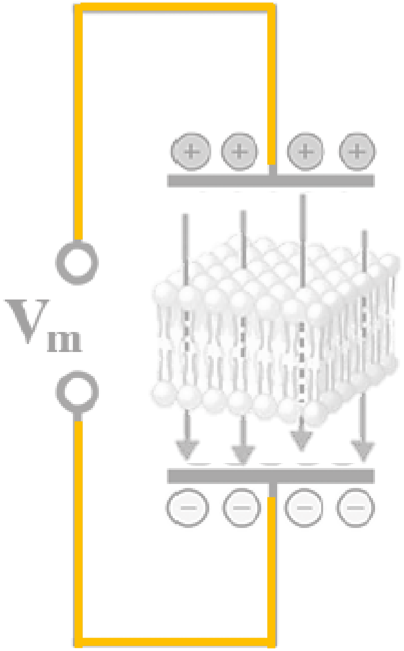

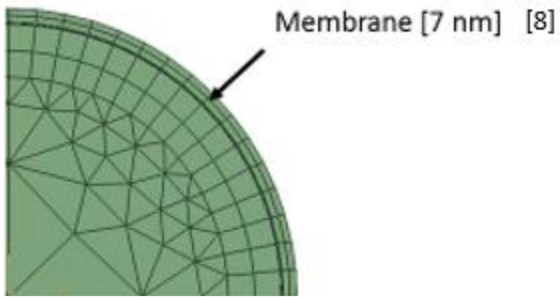

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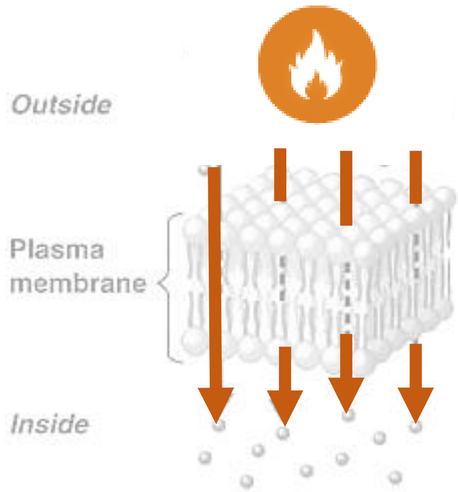

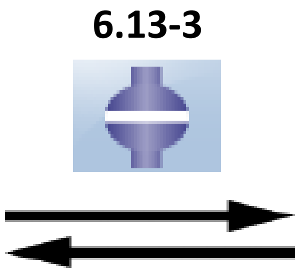

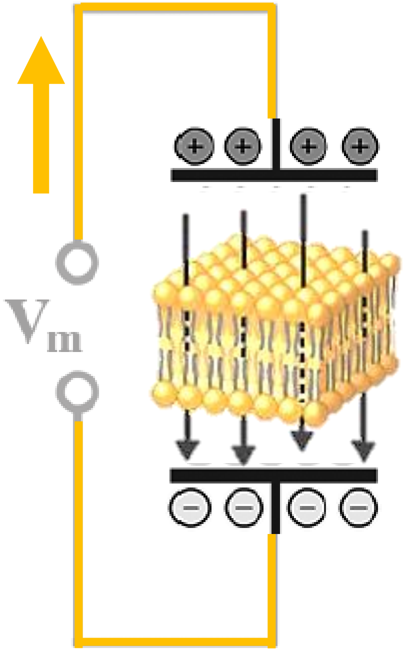

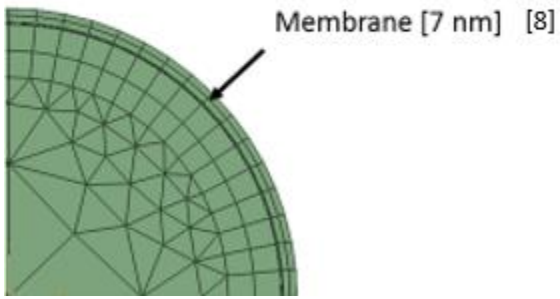

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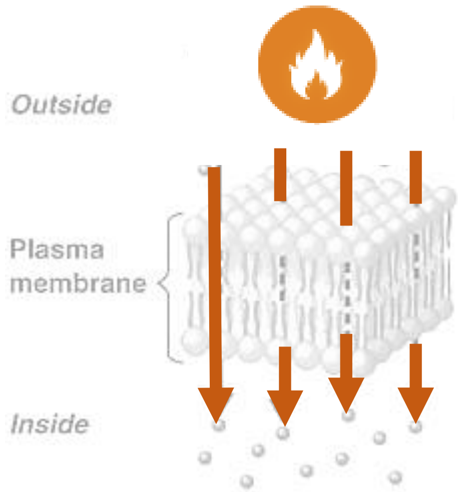
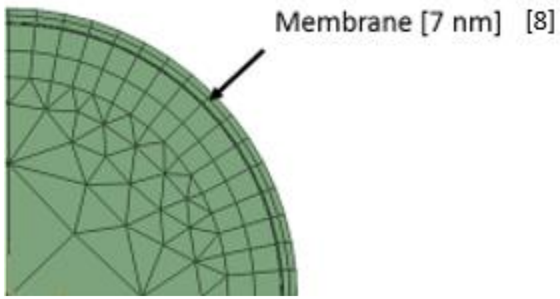


6.13-3

$$V_m > V_{Th}$$



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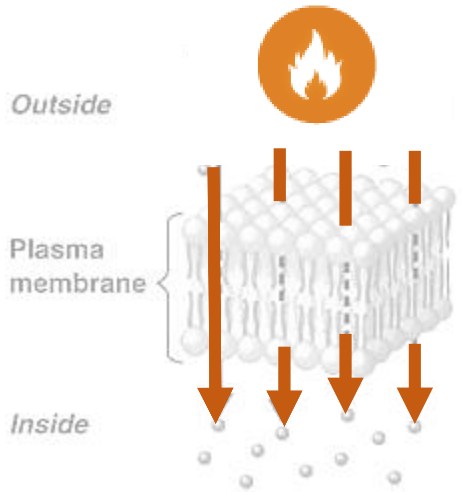
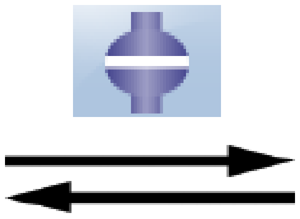
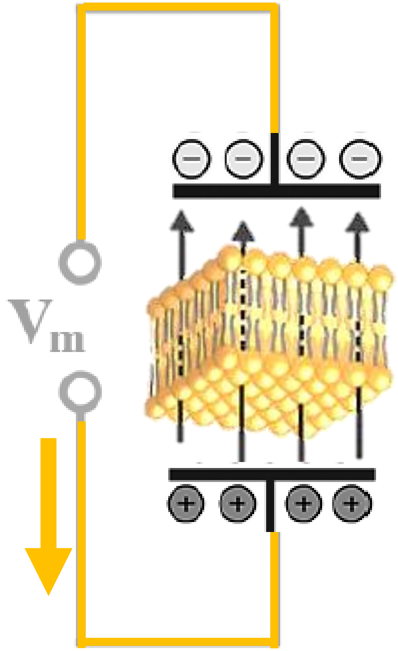
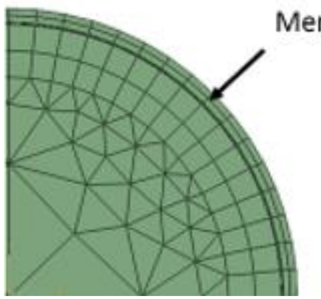






$V_m > V_{Th}$



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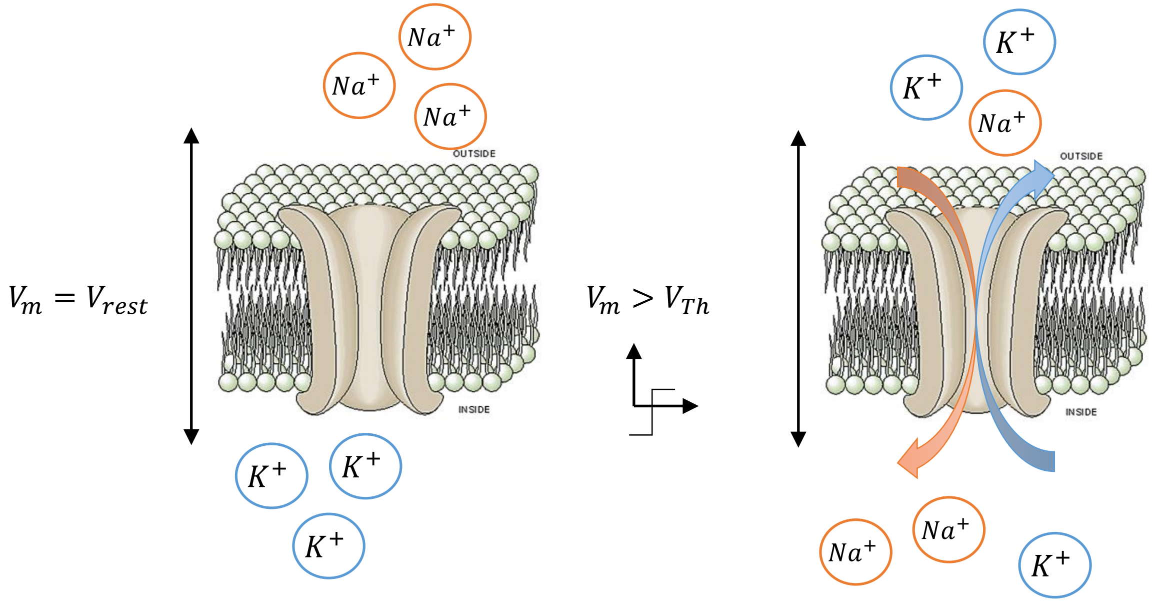

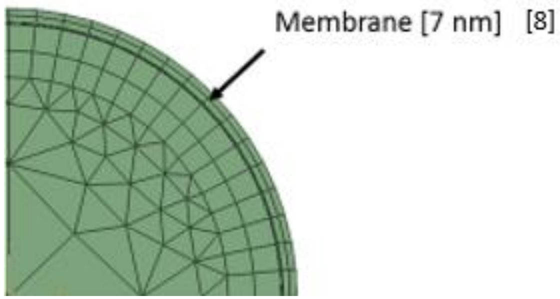

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USDFLD

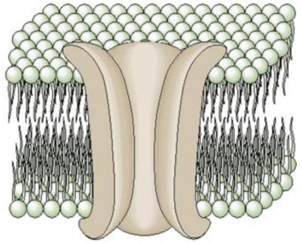
Transferring of ions cross the membrane $\{g_{Na}, g_K, g_l\}$ [2]

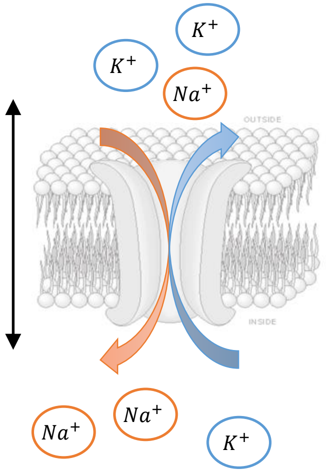
Ionic Currents $\{Na, K, l\}$ [2]

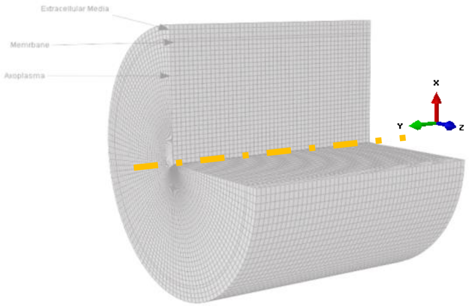
The nonlinear Cable Equation [5]

DISP

Trans-membrane voltage [6]







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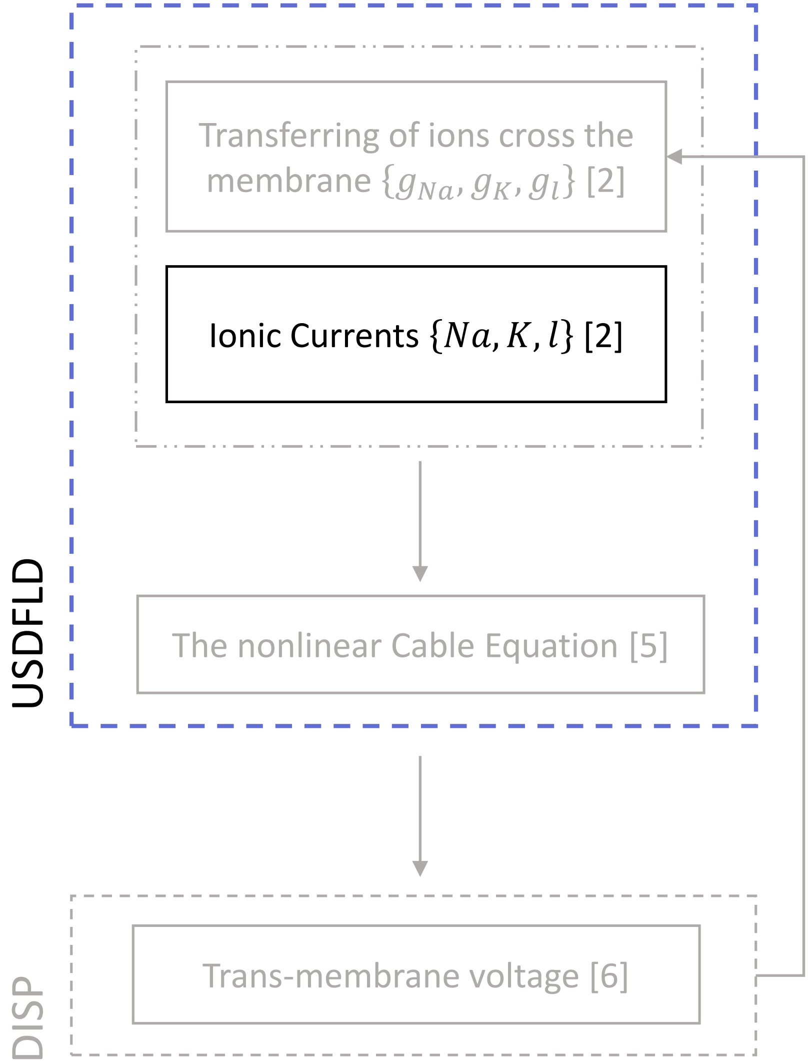

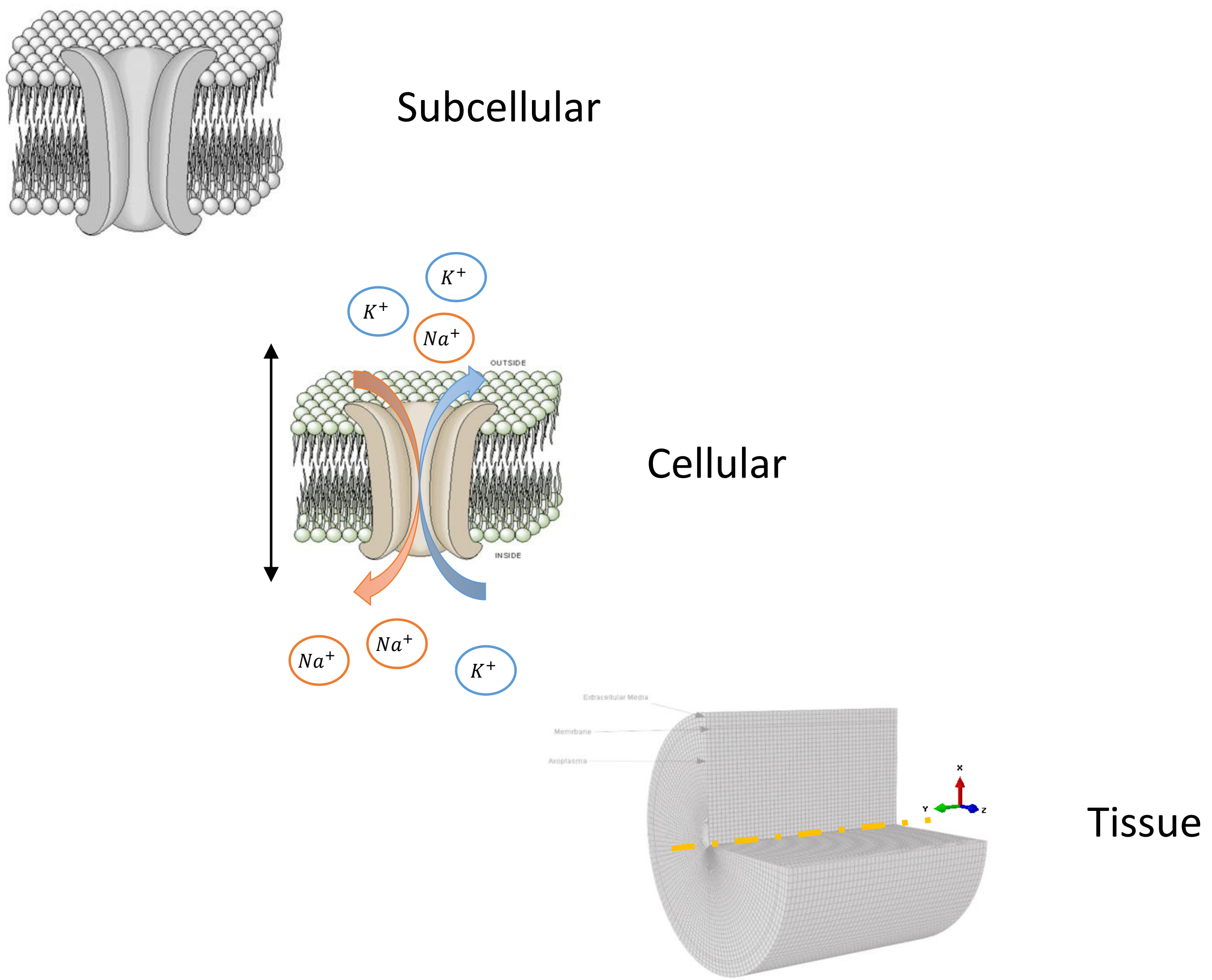

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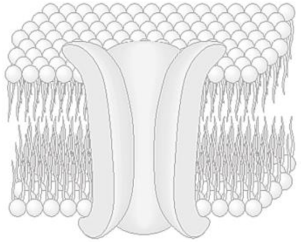
Ionic Currents $\{Na, K, l\}$ [2]

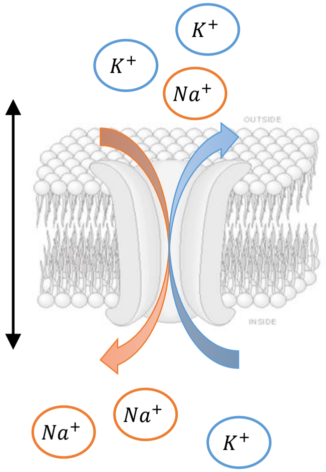
The nonlinear Cable Equation [5]

DISP

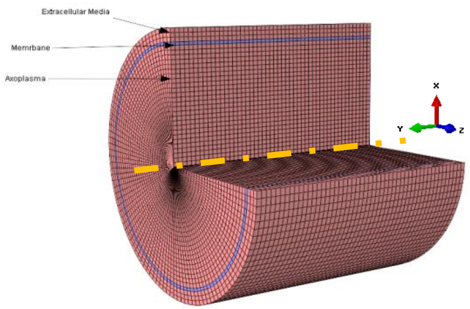
Trans-membrane voltage [6]



Subcellular



Cellular



Tissue

[2] Hodgkin, A. L., & Huxley, A. F. (1952).
[5] Halassy, S. (2012). [6] Plonsey, R., & Malmivuo, J. (1995).

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Data [8],[9]

$n = 0,1,2.$ → $n = 1.$

Analytical Solution with voltage boundary condition [8],[9]

[9] Tahayori, B., (2012).
[8] Meffin, H., (2012).

Data [8],[9]

$$\begin{cases} n = 1. \\ J(1) = 5000\ {}^{\mu A}/_{m} \\ \sigma = 10\mu m \end{cases}$$

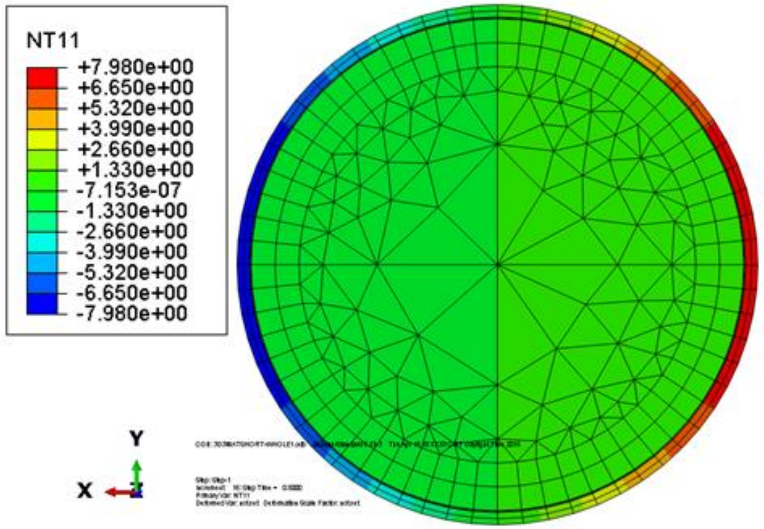
Analytical Solution with voltage boundary condition [8],[9]

$$g(z) = \frac{1}{\sqrt{2\pi}} e^{\frac{-z^2}{2\sigma^2}}$$

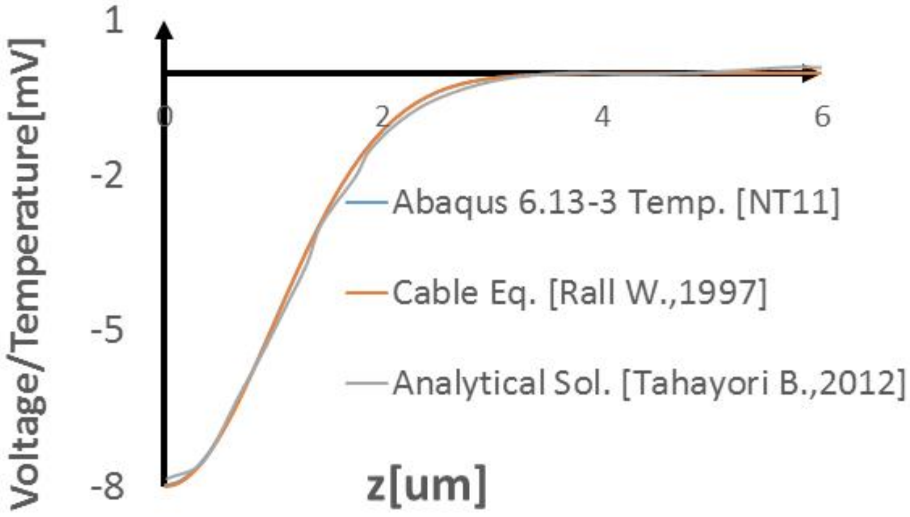
$$\boldsymbol{V_{input}\,(z, \theta, t) = -V(n)g(z)\cos(n\theta)\cos(\omega t)}$$

[9] Tahayori, B., (2012).
[8] Meffin, H., (2012).

(a)



(c)



Data [8],[9]

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$$g(z) = \frac{1}{\sqrt{2\pi}} e^{\frac{-z^2}{2\sigma^2}}$$

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$$V_M(z,\theta,t) \approx V_{input}\,(z,\theta,t)$$

[8],[9]

[9] Tahayori, B., (2012).
[8] Meffin, H., (2012).

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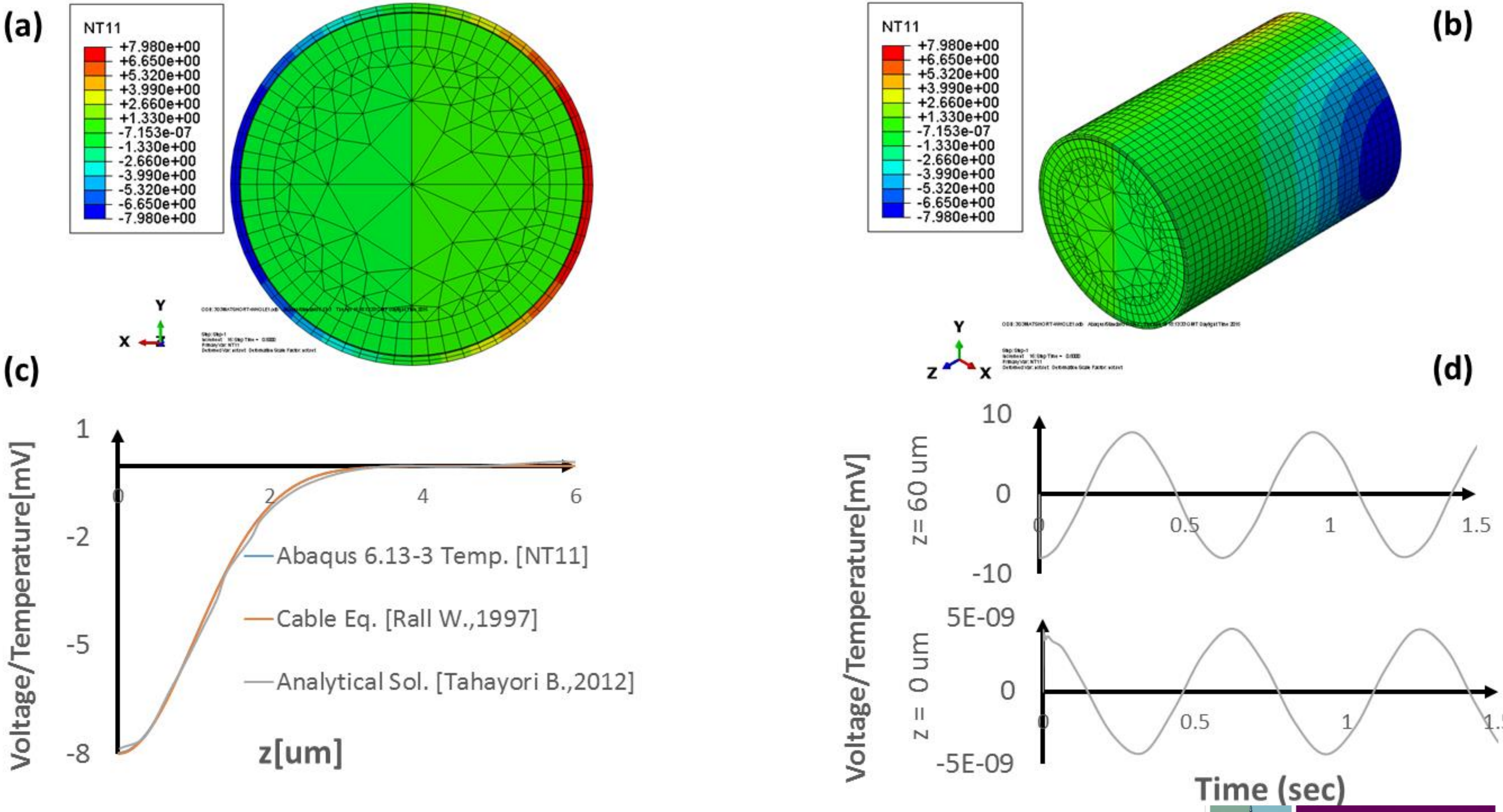
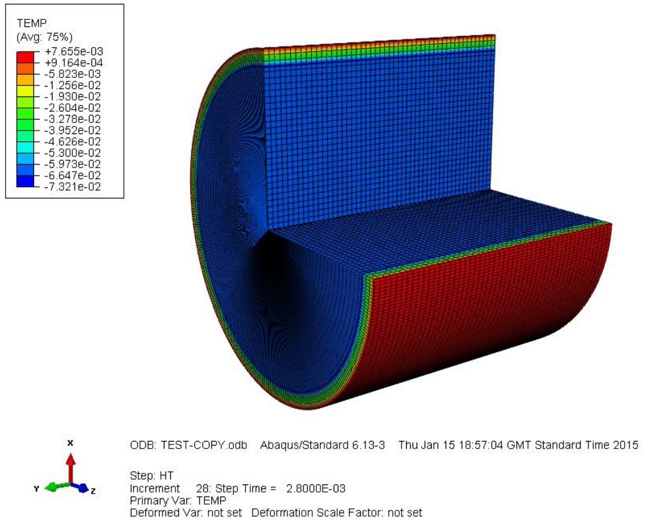
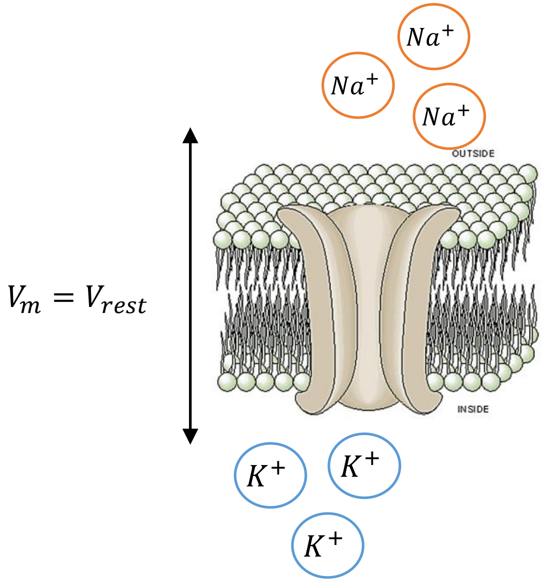
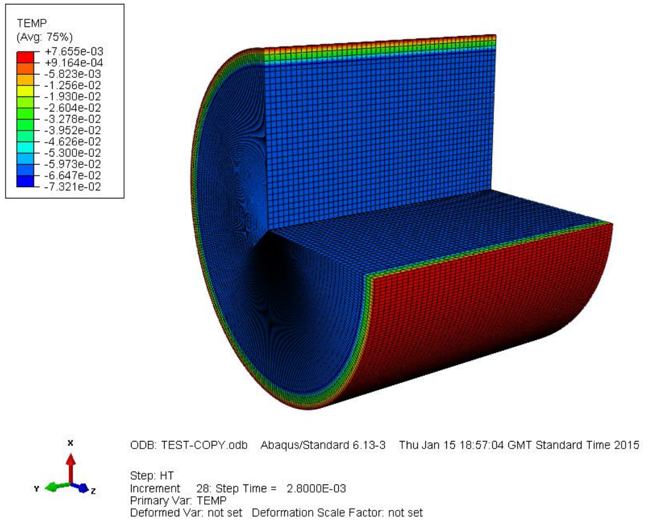


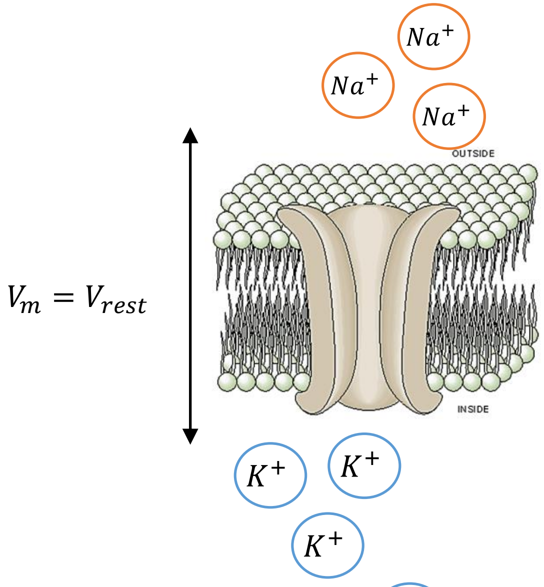
[9] Tahayori, B., (2012).
[8] Meffin, H., (2012).

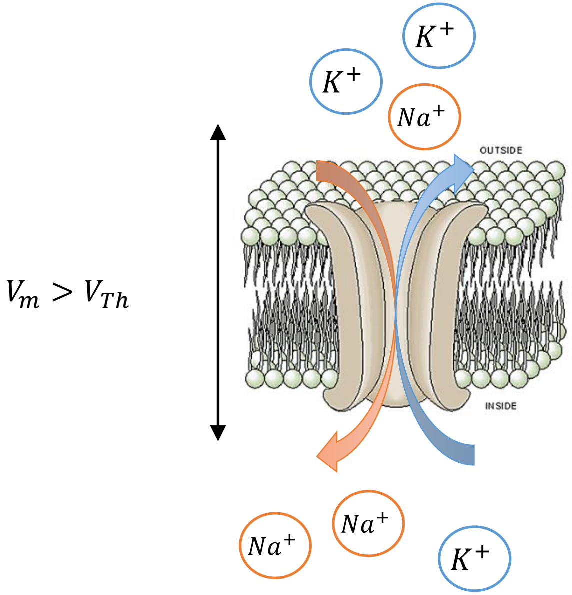


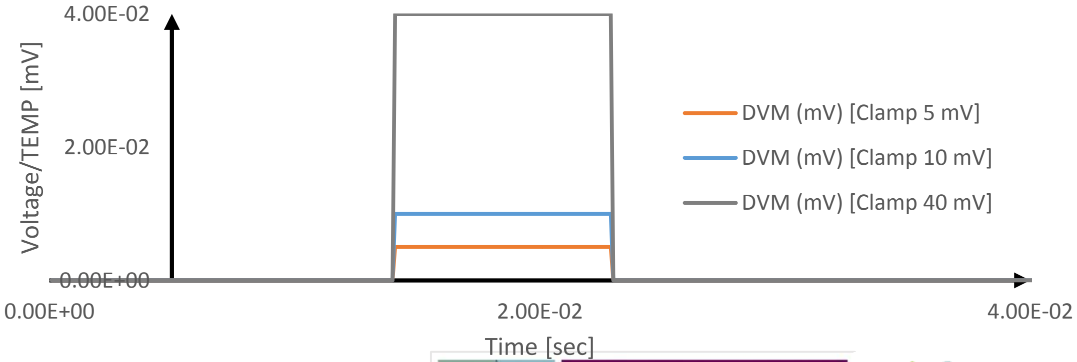


[6] Plonsey, R., & Malmivuo, J. (1995).









[2] Hodgkin, A. L., & Huxley, A. F. (1952).
[3] Plonsey, R., & Malmivuo, J. (1995).

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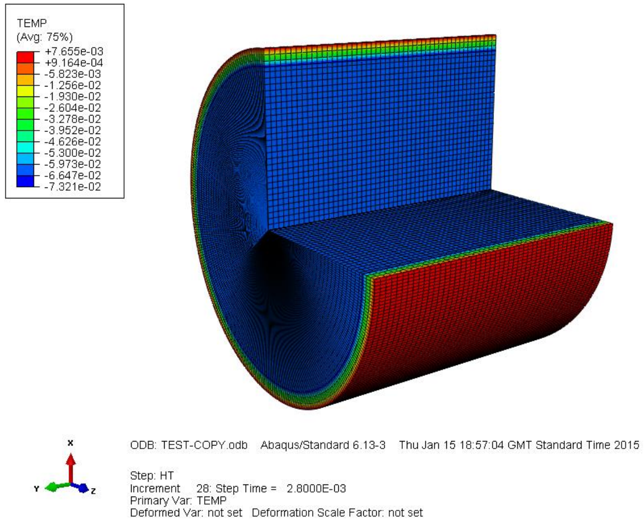

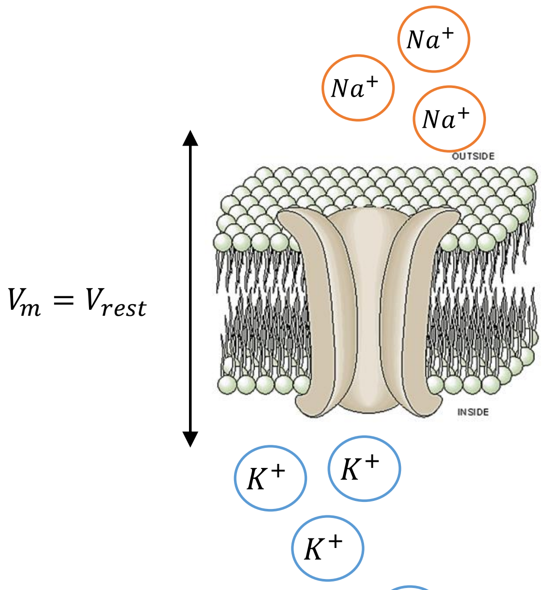

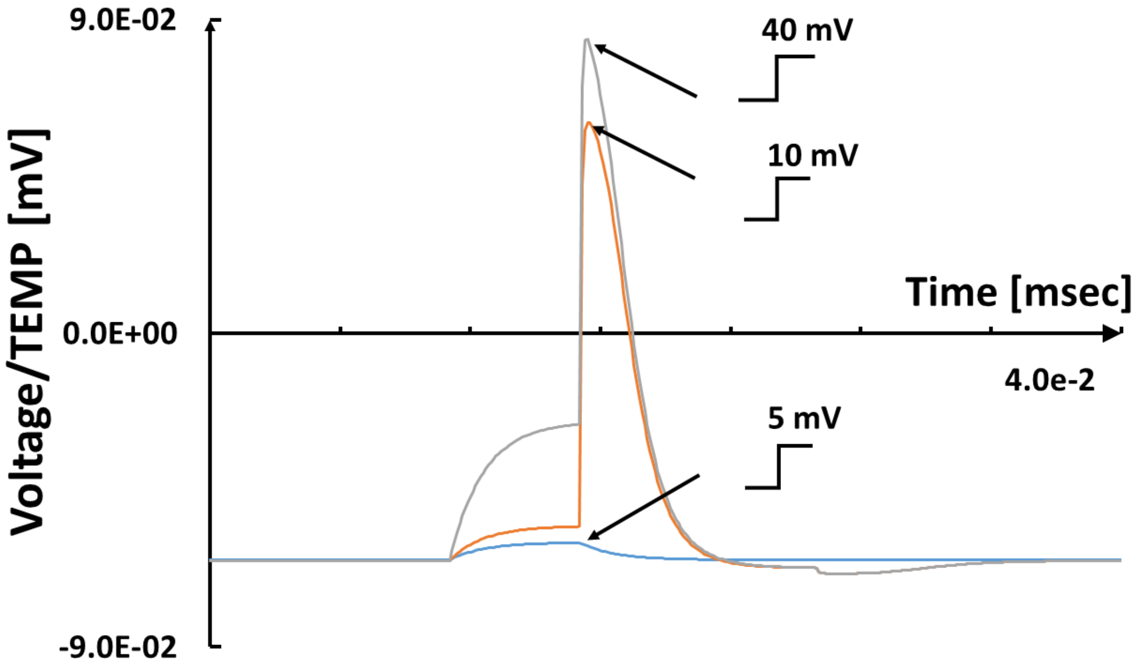

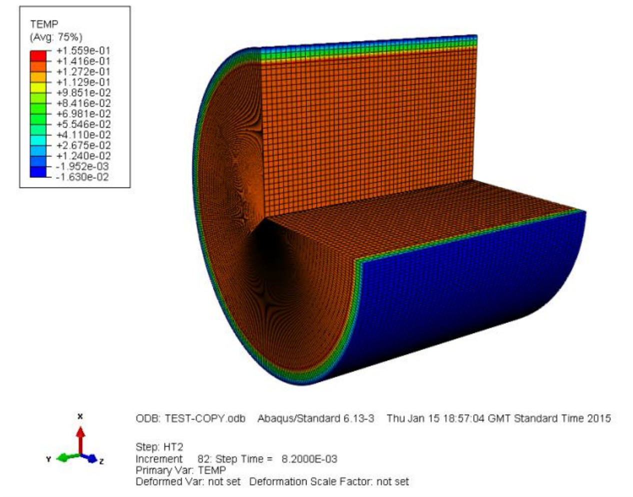

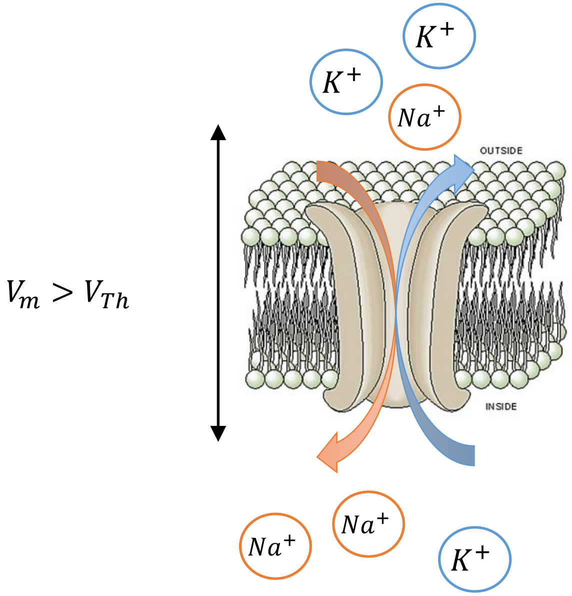

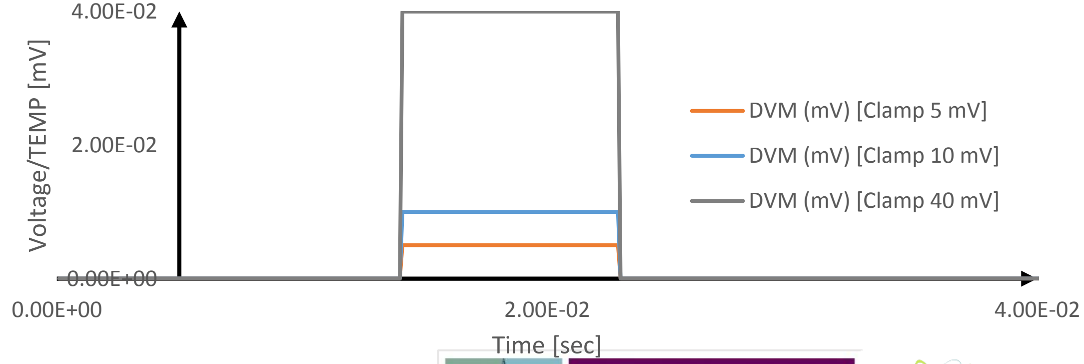

[2] Hodgkin, A. L., & Huxley, A. F. (1952).
[3] Plonsey, R., & Malmivuo, J. (1995).

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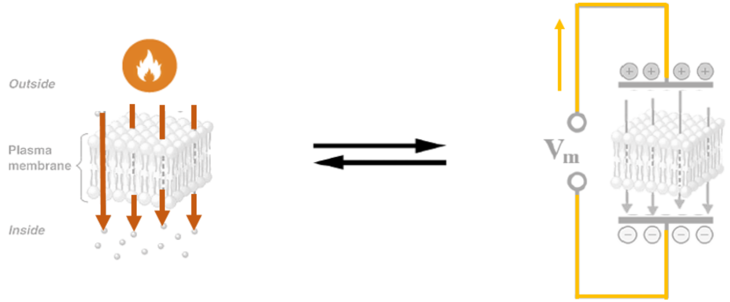
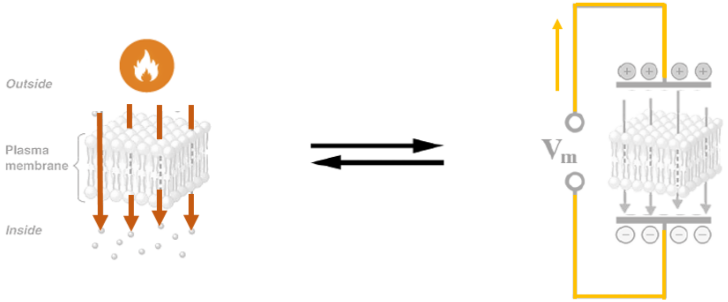


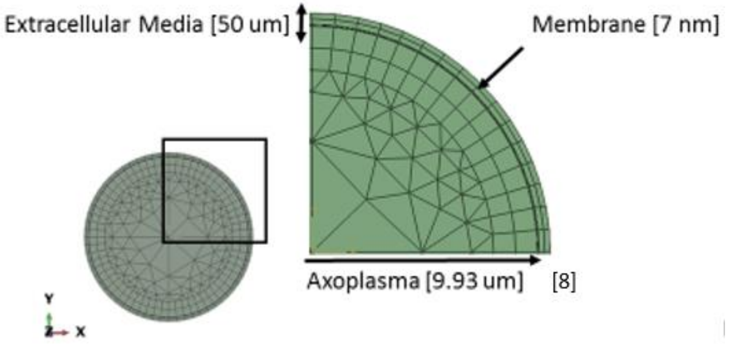
Thermo-electrical equivalents

- Cable Equation – Heat Equation

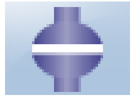


Thermo-electrical equivalents

- Cable Equation – Heat Equation

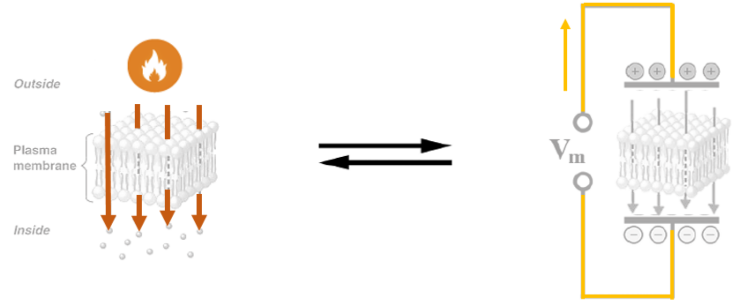


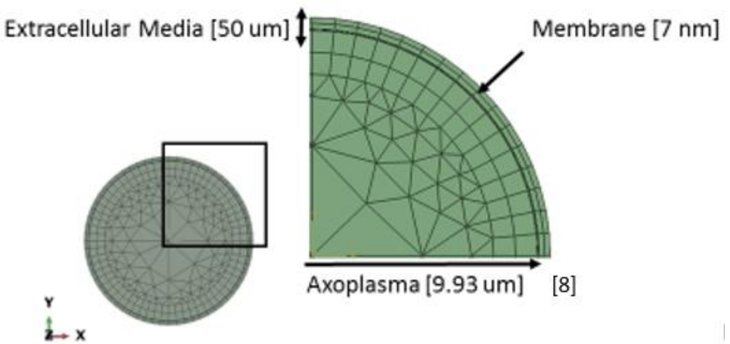
6.13-3



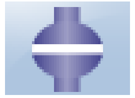
Nerve cell in FEA

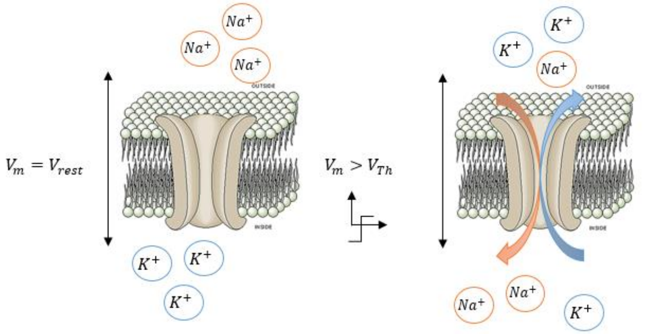
- The non-linear Hodgkin-Huxley model with gating activation





6.13-3





Thermo-electrical equivalents

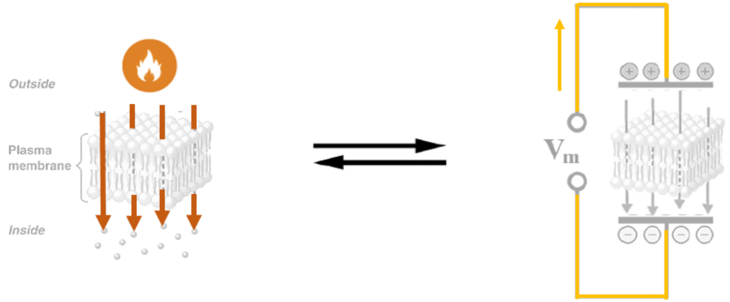
- Cable Equation – Heat Equation

Nerve cell in FEA

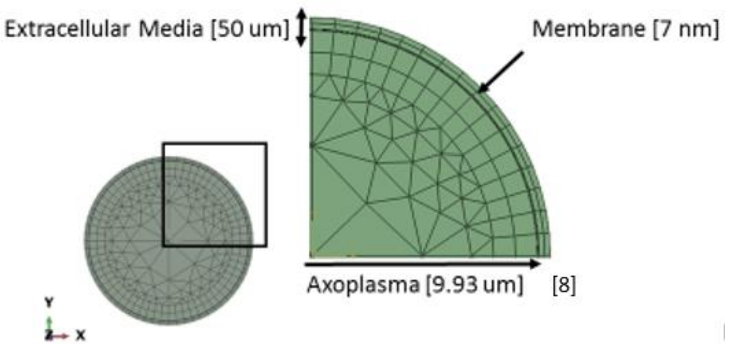
- The non-linear Hodgkin-Huxley model with gating activation

Validation with analytical solutions

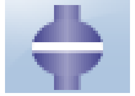
- In space and time

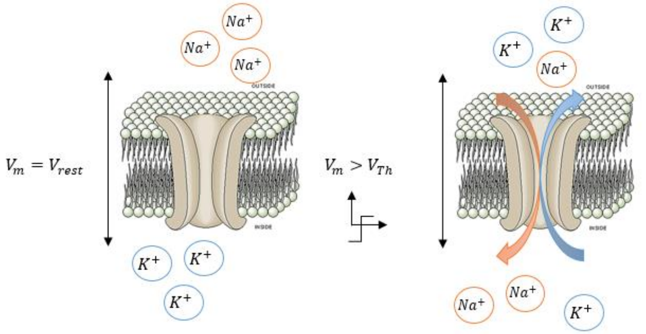


Applications



6.13-3





- Cardiac cells;
- General diffusion processes;
- Piezoelectric materials;
- Diffusive solitary wave or soliton in excitable media [10].

[10] Vargas E. V., (2011).

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Acknowledgements

McHugh Group

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Enda Boland

Mary O'Shea

Brían O'Reilly

Rosa Shine

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